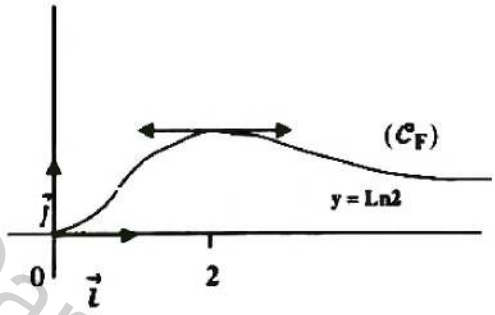
5) a)

$$F'(x) = \frac{\ln 2 - \ln x}{h(2x).h(x)} > 0$$

x	0	2	$+\infty$
$F'(x)$	+	0	-
$F(x)$	0 ↗	F(2)	↘ Ln2

b)



$F'_d(0) = 0$

III) $n \in IN^*$

1) $v_n = \int_{\frac{1}{n}}^{1} \frac{1}{t \ln t}\, dt$

a) on a : $h(t) \geq 1$; $\forall t \in]0, +\infty[$

$$\rightarrow \frac{1}{h(t)} \leq 1 \quad \rightarrow \frac{t}{h(t)} \leq t$$

$$\Rightarrow \frac{t}{t - \ln t} \leq t \quad ; \forall t \in]0, +\infty[$$

b) $v_{n+1} = \int_{\frac{1}{n+1}}^{1} \frac{t}{h(t)}\, dt$

$$v_{n+1} = \int_{\frac{1}{n+1}}^{\frac{1}{n}} \frac{t}{h(t)}\, dt + \int_{\frac{1}{n}}^{1} \frac{t}{h(t)}\, dt$$

$$v_{n+1} - v_n = \int_{\frac{1}{n+1}}^{\frac{1}{n}} \frac{t}{h(t)}\, dt$$

$v_{n+1} - v_n \geq 0$ car $\frac{t}{h(t)} \geq 0$

$\forall t \in [\frac{1}{n+1}, \frac{1}{n}]$

D'où v_n est croissante

c) pour $t \in [\frac{1}{n}, 1]$

$0 \leq \frac{t}{h(t)} \leq t$ d'après a)

$$\Rightarrow 0 \leq \int_{\frac{1}{n}}^{1} \frac{t}{h(t)}\, dt \leq \int_{\frac{1}{n}}^{1} t\, dt$$

Qarray.tn -Aqra Blech- Qarray.tn